

# Cognitive Psychology

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Lecture 12: Problem-Solving and Reasoning

# Outline for today

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- Different approaches to problem-solving
  - Gestalt approach
  - Information-processing approach
  - Using analogies
- Problem-solving & expertise
- Problem-solving & creativity (no slides / textbook only)

# What is a problem?

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- An obstacle between a present state and a goal
- Not immediately obvious how to get around the obstacle
- Difficult

# The Gestalt Approach

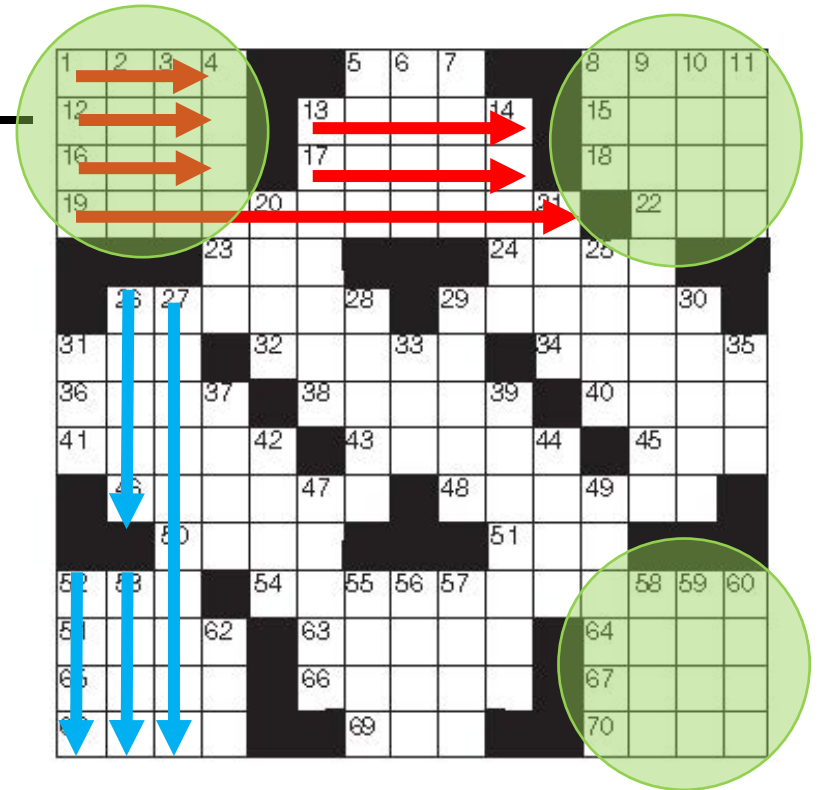
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- Problem-solving is about how we **represent** the problem in our mind
- And involves **restructuring** or reorganizing the information to reach a solution

# The Gestalt Approach

## Representation

- Many different ways to represent the same problem
- The way a problem is *presented* will impact how we **represent** it



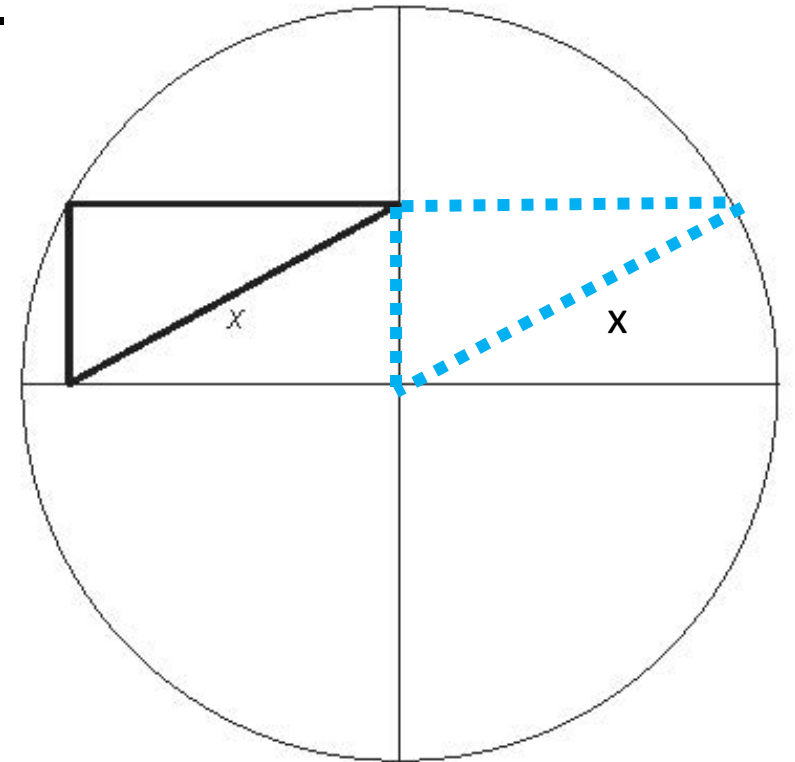
**Figure 12.1** This is a picture of how a crossword puzzle is represented on the page. In addition, there are clues for filling in the horizontal and vertical words. © Cengage Learning

# The Gestalt Approach

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## Representation

- The way we **represent** the problem can significantly impact our ability to solve it
- “If the length of the circle’s radius is  $r$ , what is the length of  $x$ ?”



**Problem:** If the length of the circle’s radius is  $r$ , what is the length of line  $x$ ?

**Figure 12.2** Circle problem. See Figure 12.26, page 367, for the solution.  
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# The Gestalt Approach

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## Restructuring

- **Insight:** Sudden realization of a problem's solution
  - Often requires restructuring the problem
  - The “aha!” moment

# The Gestalt Approach

## Are there “insight” and “non-insight” problems?

- Metcalfe and Wiebe (1987)
  - As you solve, make “warmth” judgments every 15-seconds



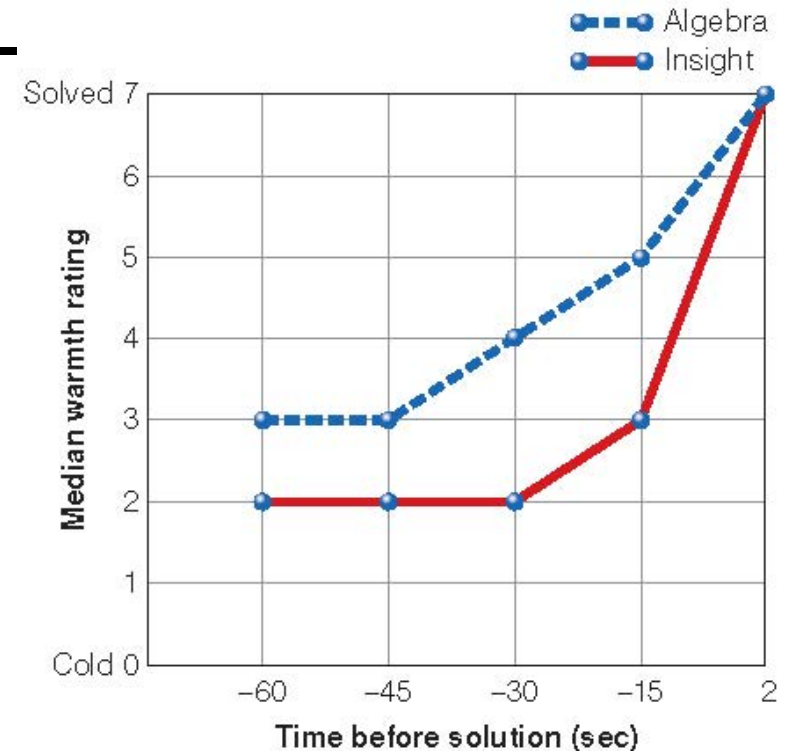
**Figure 12.3** (a) Triangle problem and (b) chain problem for “Two Insight Problems” demonstration. See page 367 for solutions. © Cengage Learning



# The Gestalt Approach

## Are there “insight” and “non-insight” problems?

- Metcalfe and Wiebe (1987)
  - Noninsight problems solved gradually
- Insight problems solved suddenly
  - Gestalts argued that this is because it requires the correct restructuring, which occurs suddenly
  - Like trying to fit a square peg into a round hole, then trying the round peg which suddenly fits



**Figure 12.4** Results of Metcalfe and Wiebe’s (1987) experiment showing subjects’ judgments of how close they were to solving insight problems and algebra problems during the minute just before solving the problem. (Source: Based on J. Metcalfe & D. Wiebe, *Intuition in insight and noninsight problem solving*, *Memory and Cognition*, 15, 238–246, 1987.)

# The Gestalt Approach

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## Obstacles to problem-solving

- The Gestalts focused on restructuring as the prime determinant or problem-solving
- Situations that prevent appropriate restructuring should produce difficult problems solved with insight
- **Fixation:** people's tendency to focus on a specific characteristic of the problem that keeps them from arriving at a solution

# The Gestalt Approach

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## Obstacles to problem-solving

- **Functional fixedness:** restricting use of an object to its familiar functions
- Example:
- **Candle problem:** Your task is to mount a candle on the corkboard so it will burn without dripping wax on the floor.
- Seeing boxes as containers inhibited using them as supports

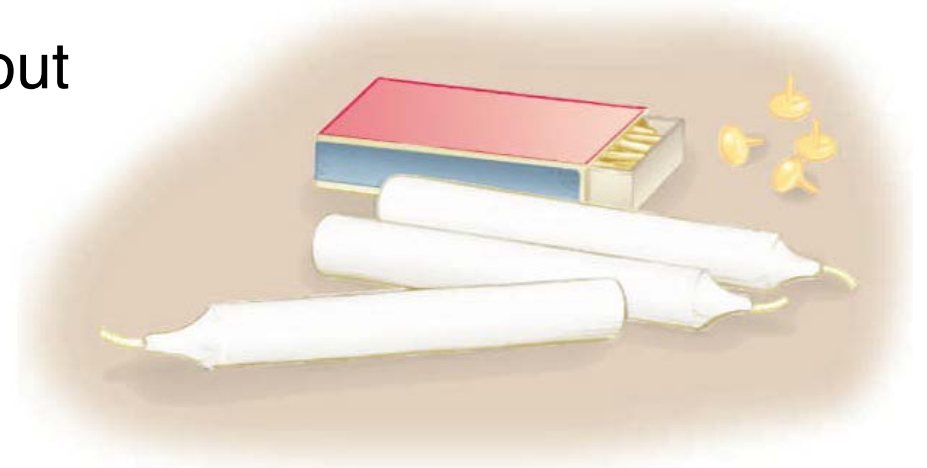


Figure 12.5 Objects for Duncker's (1945) candle problem. (Source: Based on K. Duncker, On problem solving, Psychological Monographs 58, 5, Whole No. 270, 1945.)

# The Gestalt Approach

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## Obstacles to problem-solving

- **Functional fixedness**
- Example:
- **Two-string problem:** Your task is to tie two strings together that hang from the ceiling
  - Too far to reach (even standing on the chair)
- People fail to see that the pliers can be used as a weight to tie to the end of one string and swing within reach



**Figure 12.7** Maier's (1931) two-string problem. As hard as the subject tries, he can't grab the second string. How can he tie the two strings together? (Note: Just using the chair doesn't work!) (Source: Based on N. R. F. Maier, *Reasoning in humans: II. The solution of a problem and its appearance in consciousness*, *Journal of Comparative Psychology*, 12, 181–194, 1931.)

# The Gestalt Approach

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## Obstacles to problem-solving

- **Mental set**

- A preconceived notion about how to approach a problem
- Based on a person's past experiences with the problem (or similar problems)
- Water-jug problem: given mental set inhibited participants from using simpler solution

# The Gestalt Approach

## Obstacles to problem-solving

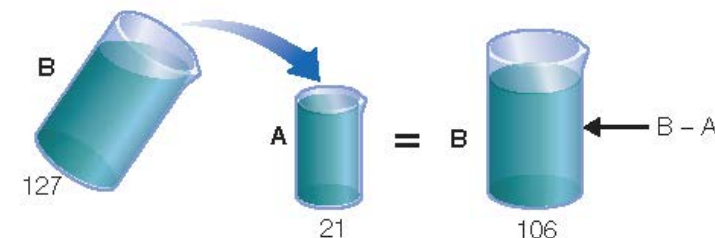
### • Mental set

- Luchins, 1942
- All problems can be solved using the formula  $B - A - 2AC$
- However, problems 7 and 8 can also be solved using the more efficient (and simpler)  $A + C$  (p. 7) and  $A - C$  (p. 8) formulas
- Participants either given problems 1-6 (mental set) or not (no mental set)
  - No mental set group: 100% use simple formulas
  - Mental set group: 23% use simple formulas

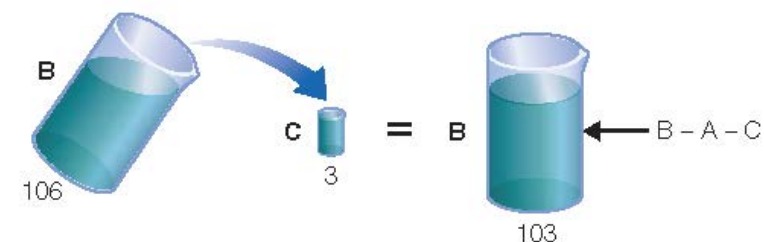
Capacities (quarts)

Problem	Jug A	Jug B	Jug C	Desired quantity
1	21	127	3	100
2	14	163	25	99
3	18	43	10	5
4	9	42	6	21
5	20	59	4	31
6	20	50	3	24
7	15	39	3	18
8	28	59	3	25

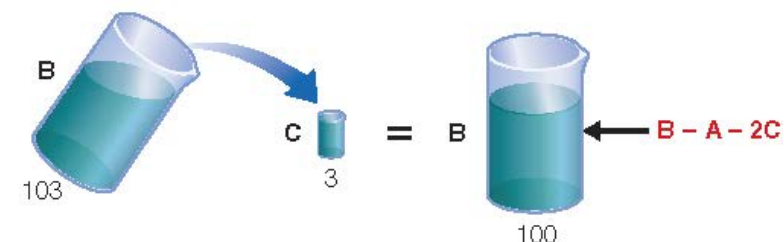
(a)



(b)



(c)



(d)

# Information-Processing Approach

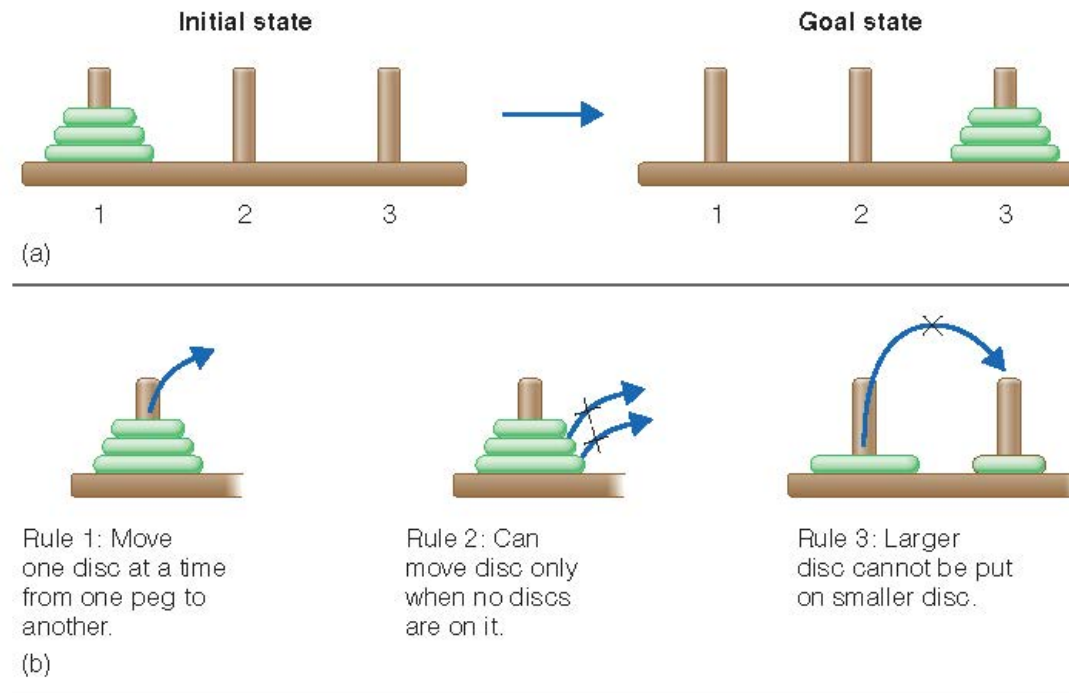
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- **Newell and Simon**

- Problem-solving is a search from the problem to the solution
  - Much like how a computer (in the 60s) would solve a problem
- We start in an **initial state** and have a **goal state** in mind
- Solving the problem involves a sequence of choices of steps, with each action creating an **intermediate state**
  - **Operators**: the actions that take us from one step to another. Usually governed by rules
- The **problem space** is all the possible states that could occur while solving a problem
- While we solve a problem, we do a **means-end analysis** to determine the actions and **subgoals** that will reduce the distance between the initial and goal states

# Information-Processing Approach

## • The Tower of Hanoi Problem



**Figure 12.9** (a) Initial and goal states for the Tower of Hanoi problem. (b) The rules for actions allowed when solving the problem. (Source: Based on K. Kotovsky, J. R. Hayes, & H. A. Simon, *Why are some problems hard? Evidence from Tower of Hanoi*, *Cognitive Psychology*, 17, 248–294, 1985.)

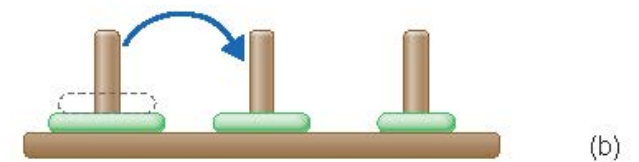
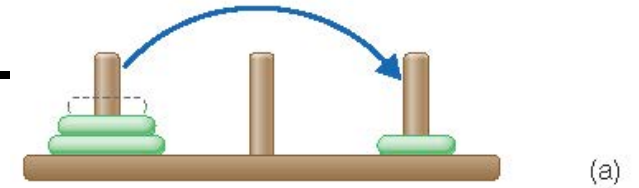


# Information-Processing Approach

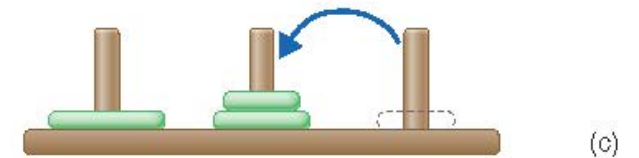
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- **The Tower of Hanoi Problem**

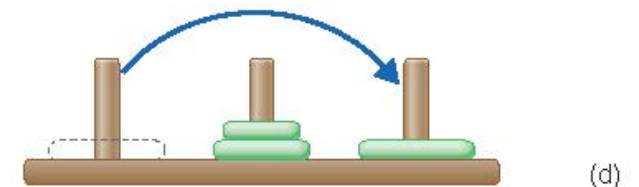
- All the steps to solving may not be clear
- However, we do a **means-end analysis** to determine **subgoals** that reduce the difference between the **initial state** and the **goal state**



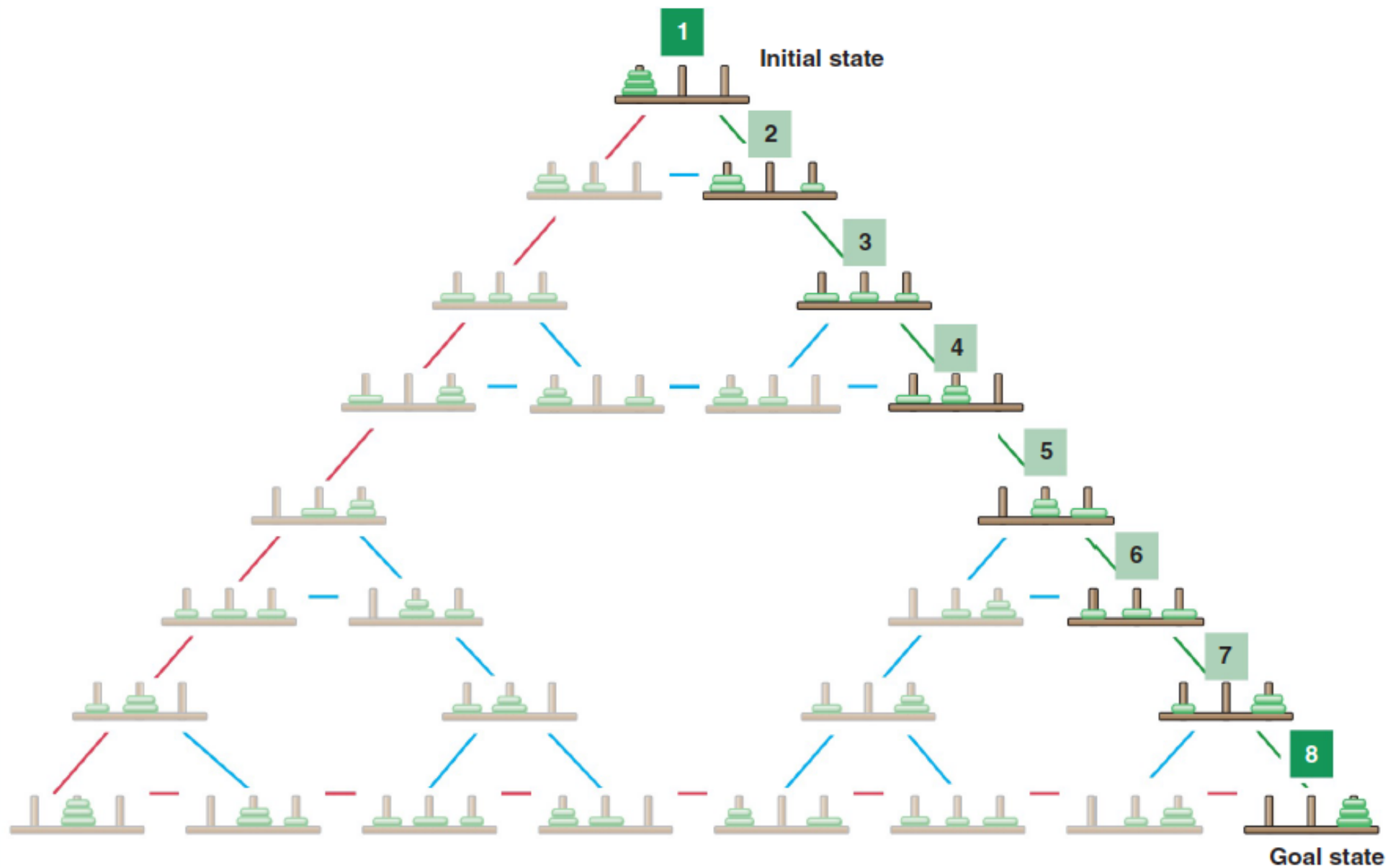
Subgoal 1: Free up large disc.



Subgoal 2: Free up third peg.



Subgoal 3: Move large disc onto third peg.



**Figure 12.10** Problem space for the Tower of Hanoi problem. The green lines indicate the shortest path between the initial state (1) and the goal state (8). The red lines indicate a longer path. (Source: Based on K. Dunbar, *Problem solving*, in W. Bechtel & G. Graham, Eds., *A companion to cognitive science*, pp. 289–298, London: Blackwell, 1998.)

# Information-Processing Approach

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- The Tower of Hanoi and the Information-processing approach was important:
  - In providing a way to formally organize a problem and specifying the problem space
  - Determining the possible pathways to a solution
  - Breaking a difficult problem down into manageable subgoals using the means-end analyses
  - Can be applied to wide range of everyday problems
- However, modern research has shown that there is more to problem-solving than specifying the problem space

# Information-Processing Approach

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## **The importance of how a problem is stated**

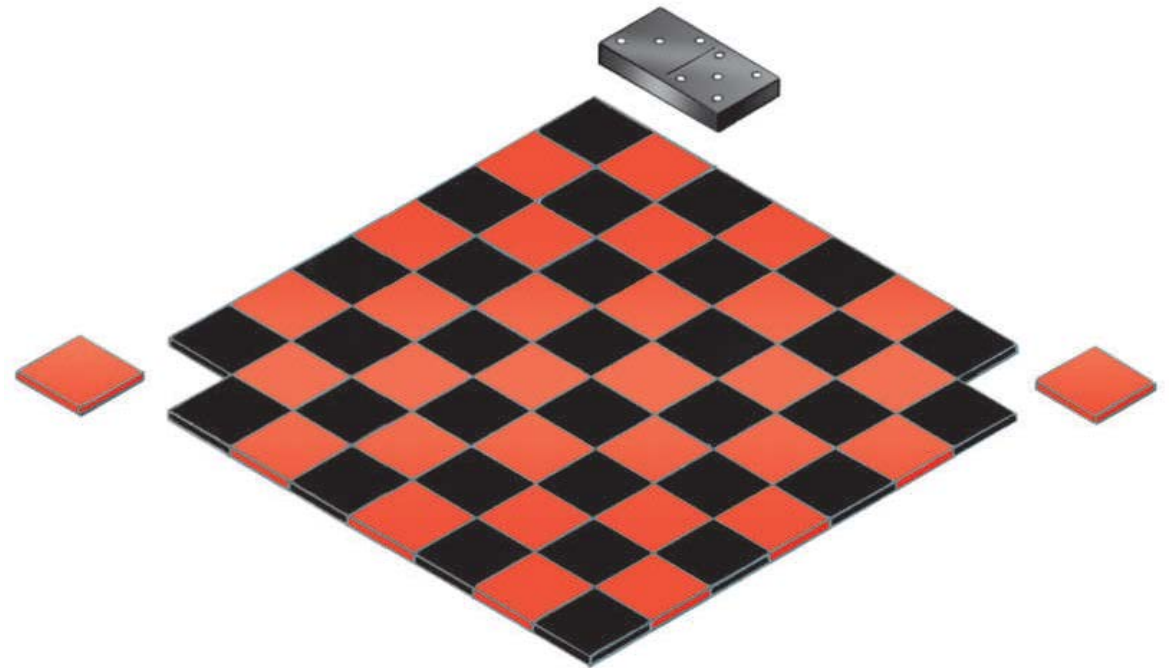
- How the problem is stated, can influence its difficulty
- **Mutilated-checkerboard problem**
  - Conditions differed in how much information provided about the squares
  - Easier to solve when information is provided that points toward the correct representation of the problem

# Information-Processing Approach

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## The importance of how a problem is stated

- **Mutilated-checkerboard problem**
- If we eliminate two corners of the checkerboard, can we now cover the remaining squares with 31 dominos?



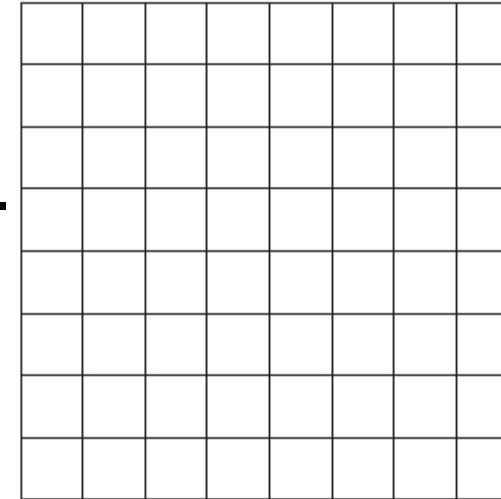
**Figure 12.13** Mutilated checkerboard problem. See demonstration for instructions.  
© Cengage Learning

# Information-Processing Approach

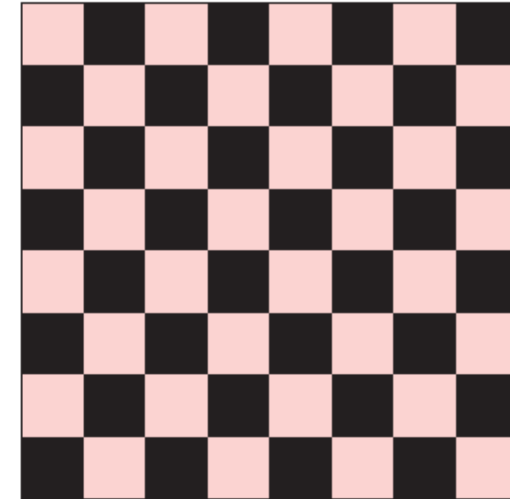
## The importance of how a problem is stated

- **Mutilated-checkerboard problem**
- If we eliminate two corners of the checkerboard, can we now cover the remaining squares with 31 dominos?
- Bread-butter group solved it twice as fast as the checkerboard
  - Same problem-space

The four conditions:



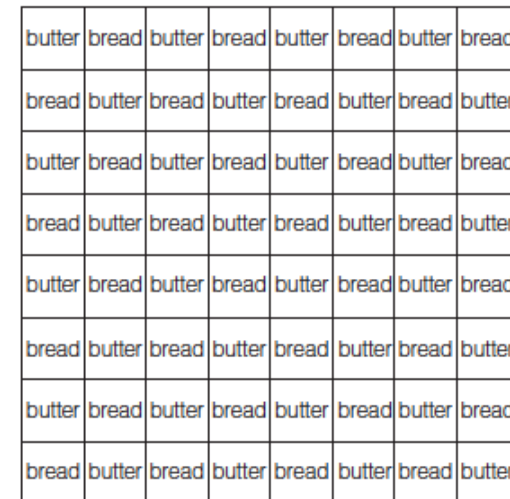
Blank



Color



Black and pink



Bread and butter

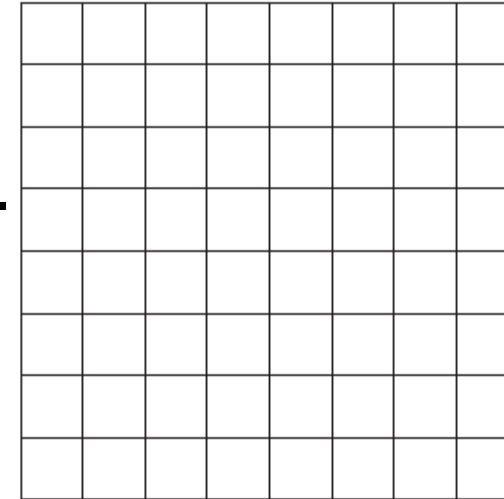
**Figure 12.14** Conditions in Kaplan and Simon's (1990) study of the mutilated checkerboard problem. (Source: C. A. Kaplan & H. A. Simon, *In search of insight*, *Cognitive Psychology*, 22, 374–419, Figure 2. Copyright © 1990 Elsevier Ltd. Reproduced with permission.)

# Information-Processing Approach

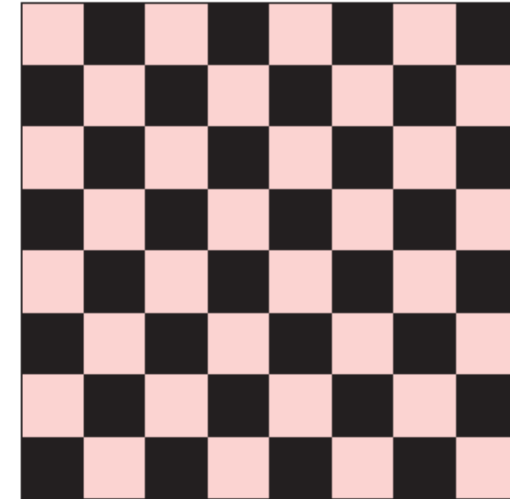
## The importance of how a problem is stated

- **Mutilated-checkerboard problem**
- The **Think-Aloud Protocol** shows that people have an “aha!” moment realizing that the bread/butter (pairs) are important
- They have to **restructure** the **representation** of the problem

The four conditions:



Blank



Color

black	pink	black	pink	black	pink	black	pink
pink	black	pink	black	pink	black	pink	black
black	pink	black	pink	black	pink	black	pink
pink	black	pink	black	pink	black	pink	black
black	pink	black	pink	black	pink	black	pink
pink	black	pink	black	pink	black	pink	black
black	pink	black	pink	black	pink	black	pink
pink	black	pink	black	pink	black	pink	black

Black and pink

butter	bread	butter	bread	butter	bread	butter	bread
bread	butter	bread	butter	bread	butter	bread	butter
butter	bread	butter	bread	butter	bread	butter	bread
bread	butter	bread	butter	bread	butter	bread	butter
butter	bread	butter	bread	butter	bread	butter	bread
bread	butter	bread	butter	bread	butter	bread	butter
butter	bread	butter	bread	butter	bread	butter	bread
bread	butter	bread	butter	bread	butter	bread	butter

Bread and butter

**Figure 12.14** Conditions in Kaplan and Simon's (1990) study of the mutilated checkerboard problem. (Source: C. A. Kaplan & H. A. Simon, *In search of insight*, *Cognitive Psychology*, 22, 374–419, Figure 2. Copyright © 1990 Elsevier Ltd. Reproduced with permission.)

# Using Analogies to Solve Problems

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- Using a solution to a similar problem guides solution to new problem
  - Russian marriage problem (source problem) → mutilated-checkerboard problem (target problem)
  - **“Analogical problem solving”**
  - **Analogical transfer**: The transfer from one problem to another
    - **Source problem to target problem**
- **Gick and Holyoak**, Using analogies requires
  - Noticing relationship
  - Mapping correspondence between source and target
  - Applying mapping



# Using Analogies to Solve Problems

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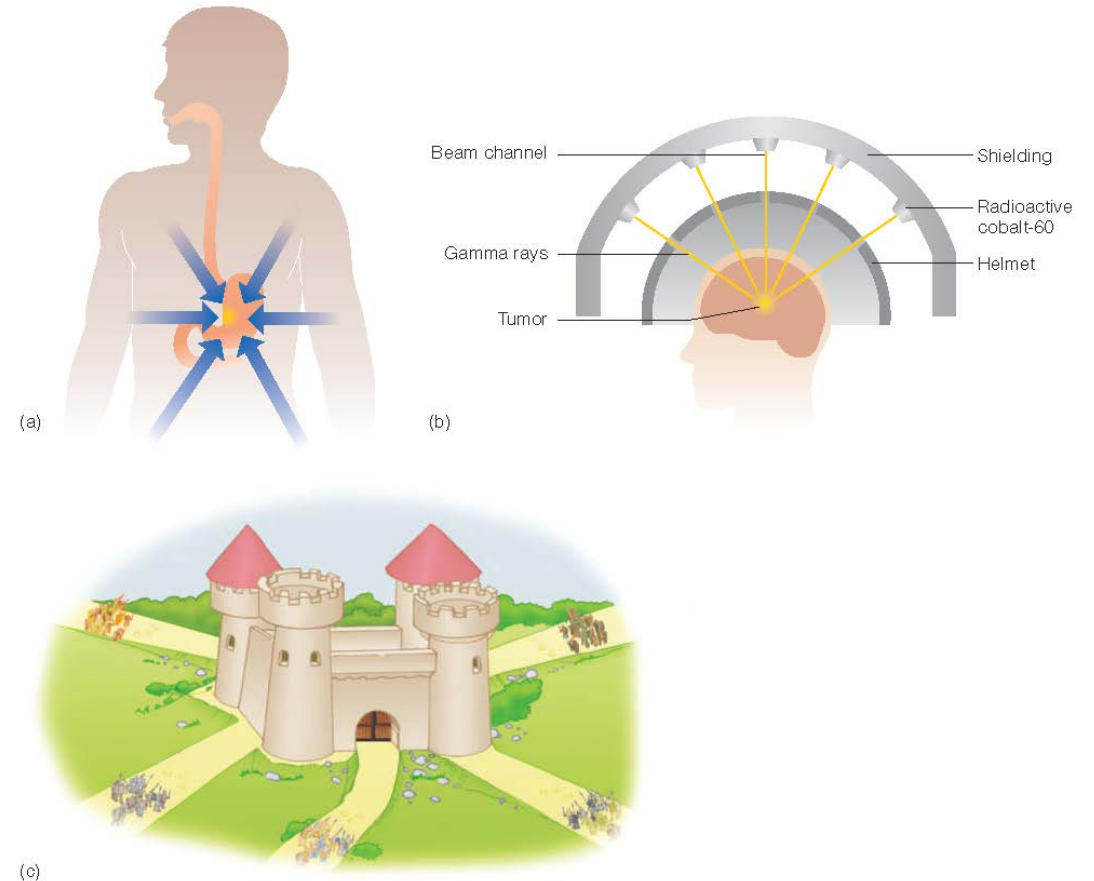
## Duncker's Radiation Problem

- Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the ray reaches the tumor at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the ray passes through on the way to the tumor will also be destroyed. At lower intensities the ray is harmless to healthy tissue, but it will not affect the tumor either. What type of procedure might be used to destroy the tumor and at the same time avoid destroying the healthy tissue (Gick & Holyoak, 1980)?

# Using Analogies to Solve Problems

## Duncker's Radiation Problem

- Difficult problem, most people didn't solve (~35% could solve)
- **Gick and Holyoak**
  - Made participants memorize the "fortress story", an analogous problem to the radiation problem and to think of that story while trying to solve the problem
  - 75% could solve the radiation problem



**Figure 12.15** (a) Solution to the radiation problem. Bombarding the tumor, in the center, with a number of low-intensity rays from different directions destroys the tumor without damaging the tissue it passes through. (b) Radiosurgery, a modern medical technique for irradiating brain tumors with a number of beams of gamma rays, uses the same principle. The actual technique uses 201 gamma ray beams. (c) How the general solved the fortress problem. © Cengage Learning

# Using Analogies to Solve Problems

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## **Duncker's Radiation Problem**

- Analogies aid problem-solving
- Often hints must be given to notice connection
  - Surface features get in the way
  - Structural features must be used
- **Gick and Holyoak**, Using analogies requires
  - Noticing relationship
  - Mapping correspondence between source and target
  - Applying mapping

# Using Analogies to Solve Problems

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## Duncker's Radiation Problem

- **Gick and Holyoak**, Using analogies requires
  - Noticing relationship
  - Mapping correspondence between source and target
  - Applying mapping
- Often hints must be given to notice connection
  - Surface features get in the way
  - Structural features must be used

# Using Analogies to Solve Problems

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## Lightbulb problem

- High surface similarities aid analogical problem solving
  - **Surface features:** Specific elements of a given problem
- Radiation problem (source problem) as analogy for the lightbulb problem (target problem)
  - 81% vs. 10% solve rate with analogous problem

# Using Analogies to Solve Problems

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## Lightbulb problem

- Making structural features more obvious aids analogical problem-solving
- **Structural features:** The underlying principle(s) that govern the solution to a problem

	LIGHTBULB PROBLEM: FRAGILE-GLASS VERSION	RADIATION PROBLEM	LIGHTBULB PROBLEM: INSUFFICIENT-INTENSITY VERSION
<b>Structural Features</b>	Laser intensity too high (breaks bulb)	Ray intensity too high (destroys tissue)	Laser intensity too low (won't fix filament)
		↔	↔
		<b>Same</b>	<b>Different</b>
		69 percent solved radiation problem	33 percent solved radiation problem

# Using Analogies to Solve Problems

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- **Analogical encoding:** the process by which two problems are compared and similarities between them are determined
  - Effective way to get participants to pay attention to structural features that aid problem-solving
- Example:
  - Teach a negotiating strategy
  - Give two examples demonstrating that strategy
    - Compare two example problems
    - Pay more attention to structural features of the problems

# Using Analogies to Solve Problems

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- **Analogical paradox**

- It can be difficult to apply analogies in the laboratory, but people routinely use analogies in real-world settings

- **In vivo problem-solving research**

- People are observed to determine how they solve problems in the real world
  - Advantage: naturalistic setting
  - Disadvantages: time-consuming, cannot isolate and control variables



# Problem-Solving & Expertise

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- What is an expert?
  - “A person who, by devoting a large amount of time to learning about a field and practicing and applying that learning, have become acknowledged as being extremely knowledgeable or skilled in that field.”
- Experts solve problems in their field faster and with a higher success rate than beginners
- Experts possess more knowledge about their fields

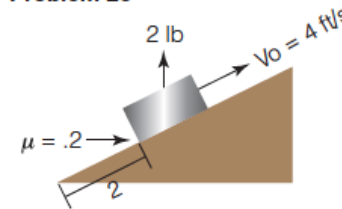
# Problem-Solving & Expertise

- Knowledge is organized so it can be accessed when needed to work on a problem
  - Novice: surface features
  - Expert: structural features

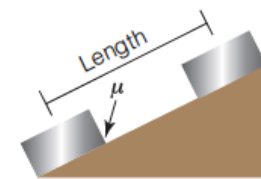
## Novice

The novice grouped problems 23 and 24 together because they both involve similar objects (inclined planes).

### Problem 23



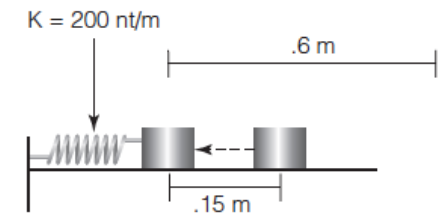
### Problem 24



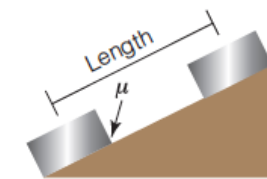
## Expert

The expert grouped problems 21 and 24 together because they both involve similar physics principles (conservation of energy).

### Problem 21



### Problem 24



**Figure 12.18** The kinds of physics problems that were grouped together by novices (left) and experts (right). (Source: Based on M. T. H. Chi, P. J. Feltovich, & R. Glaser, *Categorization and representation of physics problems by experts and novices*, *Cognitive Science*, 5, 121–152, 1981. Reprinted by permission of Taylor & Francis Group.)

# Problem-Solving & Expertise

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- Experts spend more time analyzing problem
- Experts are no better than novices when given problems outside of their field
- Experts less likely to be open to new ways of looking at problems